1. (Ex10.1) A random sample \( X_1, \ldots, X_n \) is drawn from a population with pdf
\[
f(x|\theta) = \frac{1}{2}(1 + \theta x), \quad -1 < x < 1, \quad -1 < \theta < 1.
\]
Find a consistent estimator of \( \theta \) and show that it is consistent.

2. (Ex10.3) A random sample \( X_1, \ldots, X_n \) is drawn from a population that is \( N(\theta, \theta) \), where \( \theta > 0 \).

(a) Show that the MLE of \( \theta \), \( \hat{\theta} \) is a root of the quadratic equation \( \theta^2 + \theta - W = 0 \), where 
\[
W = \frac{1}{n} \sum_{i=1}^{n} X_i^2,
\]
and determine which root equals the MLE.

(b) Find the approximate variance of \( \hat{\theta} \) using the techniques of Section 10.1.3.

3. (Ex10.9) Suppose that \( X_1, \ldots, X_n \) are iid Poisson(\( \lambda \)). Find the best unbiased estimator of

(a) \( e^{-\lambda} \), the probability that \( X = 0 \).

(b) \( \lambda e^{-\lambda} \), the probability that \( X = 1 \).

(c) For the best unbiased estimators of parts (a) and (b), calculate the asymptotic relative efficiency with respect to the MLE. Which estimators do you prefer? Why?

4. (Ex10.34) For testing \( H_0 : p = p_0 \) versus \( H_1 : p \neq p_0 \), suppose we observe \( X_1, \ldots, X_n \) iid Bernoulli(\( p \)).

(a) Derive an expression for \(-2 \log \lambda(x)\), where \( \lambda(x) \) is the LRT statistic.

(b) As in Example 10.3.2, simulate the distribution of \(-2 \log \lambda(x)\) and compare it to the \( \chi^2 \) approximation.

5. (Ex10.35) Let \( X_1, \ldots, X_n \) be a random sample from a \( N(\mu, \sigma^2) \) population.

(a) If \( \mu \) is unknown and \( \sigma^2 \) is known, show that \( Z = \sqrt{n}(\bar{X} - \mu_0)/\sigma \) is a Wald statistic for testing \( H_0 : \mu = \mu_0 \).

(b) If \( \sigma^2 \) is unknown and \( \mu \) is known, find a Wald statistic for testing \( H_0 : \sigma = \sigma_0 \).