

Computational Homework Assignment 2

The following two problems can be done in a small group of students (indicate your group's members). See below for the expected guidelines for your codes:

1. In class we describe a calibration method for the Hull-White model based on Cap prices. This problem is related to this application:

- (a) Write a program that implements the referred calibration method. That is, your program must be able to find the values of the parameters a and σ that minimize

$$\min_{a,\sigma} \sum_i \left(\frac{F_i(a,\sigma) - F_i^*}{F_i^*} \right)^2,$$

where F_i^* are the market prices of the caps and $F_i(a,\sigma)$ are the corresponding Hull-White theoretical values.

Hint: Matlab has several classical minimization routines.

- (b) Using your program in (a) and the market data of Section 7.3.4 in Clewlow and Strickland, verify (or contradict) their calibration results. Justify your choice of parameter's values.
- (c) For your calibration values, plot together the market Black volatilities and theoretical Hull-White Black volatilities against maturity. Describe your findings.

2. Implementation of the LIBOR Market Model:

- (a) Find current market data of caps in the US market.

Hint: This can be found in BLOOMBERG, which is available in one station of the management library.

- (b) Write a program that returns volatility functions $\sigma_1(\cdot), \dots, \sigma_N(\cdot)$ that match perfectly all Black's cap volatilities $\bar{\sigma}_i$ of your cap data. You need to make a structural assumption on the volatilities $\sigma_i(\cdot)$. You can assume any of the structures in page 377 of Bjork (don't use the simplest possible).
- (b) Using Monte Carlo simulation (see equation (25.36) in Björk) price a *Ratchet Cap*. In this cap contract, the cap rate at each period is set equal to the LIBOR rate at the previous reset date plus a spread s ; then, the payoff of the i th caplet (per dollar) is

$$\mathcal{X}_i = (T_i - T_{i-1})(L(T_{i-1}, T_i) - (L(T_{i-2}, T_{i-1}) + s))_+.$$

(see Section 24.3 and Table 24.3 in *Hull* 5th edition- or Section 29.3 in *Hull* 6th edition- for a numerical example).

Guidelines for the codes:

1. Your different subroutines and nontrivial procedures in your programs (or in your classes if using C++) should be properly documented.
2. In particular, this means that other different from you can understand what the input and output of each procedure is, what task(s) is performing, and what method is being used to obtain the output (describe briefly the method and/or give a reference for the method employed if it is available somewhere).