Math516/Stat541 Fall 2009

Homework assignment 2

1. Let $t < T_0 < \cdots < T_n$ be a given tenor. Consider a bond that pays a floating coupon of $c_i = (T_i - T_{i-1})L(T_{i-1}, T_i)$ at time T_i for each $i = 1, \ldots, n$. By devising a replicating trading strategy, show that the time-t price of the previous bond is $p(t, T_0)$.

2. Suppose that, at time $T_0 = 0$, there are available in the market $n - 1$ zero-coupon bonds of maturities $T_1, \ldots, T_{n-1}$ and a swap of maturity T_n and payment dates $T_1 < \cdots < T_n$. Is it possible to synthetically create a zero-coupond bond of maturity T_n using the available bonds and the swap? If yes, how?

3. (i) Deduce the time $t = 0$ fair value of a swap contract, assuming that the fixed interest payments are every 6 months, but the floating interest payments are every 3 months. (ii) Repeat the problem, assuming that there is no interest payment at the end of the first 3 months. (iii) In the situation of (ii), what is the fair value of the swap rate in terms of zero-coupon bond prices.

4. The 1-year LIBOR rate is 10%. A bank trades swaps where a fixed rate of interest is exchanged for a 12-month LIBOR with payments being exchanged annually. The 2- and 3-year swap rates are 11% and 12% per annum. By using a Bootstrap-like method, determine the 2- and 3-year LIBOR rates.

5. Consider a portfolio of n bonds with corresponding weights $\beta_1, \ldots, \beta_n$ (which can be either positive or negative depending on wether we buy or sell the bond). Let p_i be the price of the i^{th} bond ($i = 1, \ldots, n$) so that the value of the portfolio is $p = \sum_{i=1}^{n} \beta_i p_i$. Show that, when the term structure of yields is flat, the duration D of the portfolio is given by

$$D = \sum_{i=1}^{n} \beta_i D_i \frac{p_i}{p},$$

where D_i is the duration of the i^{th} bond.

6. Compute the duration of a bond with nominal value \$100 that pays a semiannual coupon rate of 6% (per annum) and has two years left until maturity. Suppose that the yield of the bond is 4%.

7. Suppose that the annual interest rate is 4%. You have a liability with a nominal value of \$300, and the payment will take place in two years. Construct a duration-immunizing portfolio that trades in two zero-coupon bonds with respective maturities of 1 year and 3 years (both with \$100 par).

   (a) How many unit of each bond should the portfolio hold;

   (b) If the rate drops 3% after one year, how much is the value of all your positions at that time?

8. The following parts can be done in a small group of students (indicate your group's member):

   (a) Collect current data from treasury bonds and swaps (you need to indicate your source and data information).

   (b) Using bootstrap and interpolation methods, strip the zero rates implied from your data.

   (c) Find a smooth interpolation of the zero-bond prices (e.g. you can use cubic splines).

   (d) Determine the instantaneous forward rate curve $f(t, T)$.