1. **(Put/Call Parity)** The goal is to show the validity of the parity formula:

\[ C_0(K, T) - P_0(K, T) = S_0 - K p(0, T), \]  

where \( p(0, T) \) is the time-0 price of a zero coupon bond with expiration \( T \) (\( T \)-bond for simplicity), while \( C_0(K, T) \) and \( P_0(K, T) \) are the time-0 prices of European call and put options with strike \( K \) and expiration \( T \) written on an underlying asset with time-0 price \( S_0 \).

(a) Consider two portfolios: (1) the first consist of a long position on the call and short position on the put; (2) the second takes a long position on the stock and sells \( K \) units of the \( T \)-bonds (hence, borrows \( K p(0, T) \) dollars). Determine the net loss/profit at time 0 generated when creating each portfolio.\(^1\)

(b) Determine the value (or payoff) of the two portfolios at expiration \( T \). What can you deduce about the costs/profits of each portfolio in part (a)?

(c) Suppose now that the stock pays discrete dividends. More specifically, suppose that if we own the stock at time 0, we will receive the dividend amounts \( D_1, \ldots, D_n \) at times \( 0 < T_1 < \cdots < T_n \leq T \), respectively. What is the corresponding put/call parity in this case?

2. The **straddle** is a European contingent claim with the following time-\( T \) payoff:

\[
\mathcal{X} = \begin{cases} 
K - S(T), & \text{if } 0 < S(T) \leq K \\
S(T) - K, & \text{if } S(T) > K .
\end{cases}
\]

This contract can be replicated\(^2\) using a **static**\(^3\) portfolio or trading strategy, consisting solely of \( T \)-zero coupon bonds, stock, and European call options.

(a) Determine the above static replicating strategy.

(b) What would the holder of this claim like to happen with the price of the underlying?

3. Consider now the **bull spread**, a European contingent claim with the following time-\( T \) payoff:

\[
\mathcal{X} = \begin{cases} 
B, & \text{if } S(T) > B, \\
S(T), & \text{if } A \leq S(T) \leq B \\
A, & \text{if } S(T) < A.
\end{cases}
\]

\((A < B \text{ known constants})\).

\(^1\)Note that this net loss/profit is the negative of the “value” of the portfolio; i.e., the profit/loss obtained when liquidating the portfolio.

\(^2\)A portfolio replicates a contingent claim if the payoff of the claim coincides with the value of the portfolio at expiration.

\(^3\)That is, all transactions in the static portfolio take place at time \( t = 0 \), after which the portfolio can’t be rebalanced.
(a) Determine the static replicating strategy, consisting solely of $T$-zero coupon bonds, stock, and European call options of strike $A$ and $B$.

(b) What would the holder of this claim like to happen with the price of the underlying?