Math515/Stat540 Spring 2009
Homework assignment 1 Figueroa-Lopez

1. The *straddle* is a European contingent claim with the following time-T payoff:

$$\mathcal{X} = \begin{cases} K - S(T), & \text{if } 0 < S(T) \le K \\ S(T) - K, & \text{if } S(T) > K. \end{cases}$$

This contract can be replicated using a *static*[1] portfolio or trading strategy, consisting solely of T-zero coupon bonds, stock, and European call options.

   (a) Determine the *static* replicating strategy.

   (b) What would a buyer of the this contingent claim like to happen with the price of the *underlying*?

2. Consider now the *bull spread*, a European contingent claim with the following time-T payoff:

$$\mathcal{X} = \begin{cases} B, & \text{if } S(T) > B, \\ S(T), & \text{if } A \le S(T) \le B \\ A, & \text{if } S(T) < A \end{cases}$$

($A < B$ known constants).

   (a) Determine the *static* replicating strategy, consisting solely of T-zero coupon bonds, stock, and European call options.

   (b) What would a buyer of the this contingent claim like to happen with the price of the *underlying* and what are the advantages of this contract over a *forward contract* on the asset?

3. Consider a one-period model with one bond and one stock. The rate of return of the bond is $R = 1/9$. The stock has initial value $S_0 = 5$ and can take up to two states at $t = 1$:

$$S_1(u) = \frac{20}{3}, \quad S_1(d) = \frac{40}{9}.$$

   (a) Find a hedging strategy for a put option on the stock with strike price $K = 5$.

   (b) What is the arbitrage-free price Π_0 of the option in (a)?

   (c) Determine a risk-neutral probability measures Q on the sample space $\Omega = \{u, d\}$; that is, Q is such that the expected return in the stock is the same as R:

$$R = E^Q\left\{\frac{S_1 - S_0}{S_0}\right\},$$

   or equivalently, the *martingale property*:

$$E^Q\left\{\frac{1}{1+R} S_1\right\} = S_0.$$

[1] that is, all transaction in the *static portfolio* or *strategy* take place at $t = 0$.

(d) Check that the arbitrage-free price Π_0 is the risk-neutral expected payoff of the put option:

$$\Pi_0 = E^Q \left\{ \frac{\mathcal{X}}{1+R} \right\}.$$

4. Consider a one-period model with one bond and a stock. The rate of return of the bond is $R = 1/9$. The stock has initial value $S_0 = 5$ and can take up to three states at $t = 1$:

$$S_1(\omega_1) = \frac{20}{3}, \quad S_1(\omega_2) = \frac{40}{9}, \quad S_1(\omega_3) = \frac{30}{9}.$$

(a) Is the market arbitrage-free?

(b) Is the market complete?

(c) Let $\mathcal{X}$ be a contingent claim. Find a linear equation in $\mathcal{X}(\omega_1), \mathcal{X}(\omega_2), \mathcal{X}(\omega_1)$ for the claim to be replicable by a trading strategy consisting of bonds and stock.