Statistical Design and Analysis of RNA-Seq Data

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ABSTRACT

Next-generation sequencing technologies are quickly becoming the preferred approach for characterizing and quantifying entire genomes. Even though data produced from these technologies are proving to be the most informative of any thus far, very little attention has been paid to fundamental design aspects of data collection and analysis, namely sampling, randomization, replication, and blocking. We discuss these concepts in an RNA-Sequencing framework. Using simulations we demonstrate the benefits of collecting replicated RNA-Sequencing data according to well known statistical designs that partition the sources of biological and technical variation. Examples of these designs and their corresponding models are presented with the goal of testing differential expression.
Next-generation sequencing (NGS) has emerged as a revolutionary tool in genetics, genomics, and epigenomics. By increasing throughput and decreasing cost, compared to other sequencing technologies (Hayden 2009), NGS has enabled genome-wide investigations of various phenomena, including single nucleotide polymorphisms (Craig et al. 2008), epigenetic events (Park 2009), copy number variants (Alkan et al. 2009), differential expression (Bloom et al. 2009), and alternative splicing (Sultan et al. 2008). One application with demonstrated effectiveness over previous technologies (e.g., microarrays and Serial Analysis of Gene Expression (SAGE)) is called RNA-Sequencing (RNA-Seq) (Cloonan et al. 2009). RNA-Seq uses NGS technology to sequence, map, and quantify a population of transcripts (Mortazavi et al. 2008; Morozova et al. 2009). While RNA-Seq is a relatively new method, it has already provided unprecedented insights into the transcriptional complexities of a variety of organisms, including yeast (Nagalakshmi et al. 2008), mice (Mortazavi et al. 2008), Arabidopsis (Eveland et al. 2008), and humans (Sultan et al. 2008).

At present, there are three widely accepted commercially available NGS devices (Illumina’s Genome Analyzer, Applied Biosystems’ SOLiD, and the 454 Genome Sequencer FLX) for RNA-Seq (Marioni et al. 2008; Cloonan et al. 2008; Eveland et al. 2008). Across platforms, the RNA-Seq methodology is approximately the same. Briefly, RNA is isolated from cells, fragmented at random positions, and copied into complementary DNA (cDNA). Fragments meeting a certain size specification (e.g., 200–300 bases long) are retained for amplification using Polymerase Chain Reaction (PCR). After amplification, the cDNA is sequenced using NGS; the resulting reads are aligned to a reference genome, and the number of sequencing reads mapped to each gene in the reference is tabulated. These gene counts, or Digital Gene
Expression (DGE) measures, can be transformed and used to test differential expression (see Morozova et al. 2009 for a review of these technologies as applied to RNA-Seq).

Although there are many steps in this experimental process that may introduce errors and biases, RNA-Seq has been hailed as the future of transcriptome research (Shendure 2008) because it potentially generates an unlimited dynamic range, provides greater sensitivity than microarrays, is able to discriminate closely homologous regions, and does not require a priori assumptions about regions of expression (Cloonan et al. 2009; Morozova et al. 2009). As research transitions from microarrays to sequencing-based approaches, it is essential that we revisit many of the same concerns that the statistical community had at the beginning of the microarray era (Kerr and Churchill 2001a).

Soon after the introduction of microarrays (Schena et al. 1995), a series of papers was published elucidating the need for proper experimental design (Kerr et al. 2000; Lee et al. 2000; Kerr and Churchill 2001a; Kerr and Churchill 2001b; Churchill 2002). All of these papers rely heavily on the three fundamental aspects of sound experimental design formalized by R. A. Fisher (1935a) seventy years ago, namely replication, randomization, and blocking. These concepts can be understood by considering the following controlled experiment that is designed to test the effectiveness of two different diets. A sound experimental design would include many different subjects (i.e., replication) recruited from multiple weight loss centers (i.e., blocking). Each center would randomly assign their subjects to one of the two diets (i.e., randomization).

Although the principles of good design are straightforward, their proper implementation often requires significant planning and statistical expertise. To date, many NGS applications, specifically RNA-Seq, have neglected good design. While a few RNA-Seq studies have reported highly reproducible results with little technical variation (e.g., Marioni et al. 2008; Mortazavi et
al. 2008), in the absence of a proper design, it is essentially impossible to partition biological variation from technical variation. When these two sources of variation are confounded there is no way of knowing which source is driving the observed results. No amount of statistical sophistication can separate confounded factors after data have been collected.

Generally, for differential expression analyses, researchers are interested in comparisons across treatment groups in the form of contrasts or pair-wise comparisons, and the designs for these analyses are usually quite simple. The good news for NGS technologies is that certain properties of the platforms can be leveraged to ensure proper design. One such feature, available in all three NGS devices, is the capacity to bar-code. Genomic fragments can be labeled or bar-coded with sample-specific sequences that in turn allow multiple samples to be included in the same sequencing reaction (i.e., multiplexing) while maintaining, with high fidelity, sample identities downstream (Craig et al. 2008; Hamaday et al. 2008; http://www3.appliedbiosystems.com/). To date, bar-coding has only been appreciated as a means to increase the number of samples per sequencing run. Yet here, we demonstrate how multiplexing can be used as a quality control feature that offers the flexibility to construct balanced and blocked designs for the purpose of testing differential expression.

We anticipate that the progression from the current un-replicated unblocked designs to more complex designs will be swift once the full offerings of NGS technologies are appreciated. Toward this end, we provide a brief review of some powerful statistical techniques for testing differential expression under a variety of designs. Although the designs that are presented are specific to RNA-Seq using the Illumina (Solexa) platform, the same statistical principles are applicable to the other NGS devices, as well as other types of comparative genetic and `omic data.
REPLICATION

**Un-replicated data:** Observational studies with no biological replication are common in the RNA-Seq literature (e.g., Marioni *et al.* 2008). In an observational study, as opposed to a controlled experiment, the assignment of subjects to treatment groups is not decided by the investigator. In many cases, the different treatment groups consist of different tissue types. For example, in Marioni *et al.* (2008) messenger RNA (mRNA) was isolated from liver and kidney tissues, randomly fragmented, and sequenced using the Illumina Genome Analyzer (GA). The Illumina technology (aka “Solexa”) relies on a flow-cell with eight lanes, or channels, and massively parallel sequencing by synthesis to simultaneously sequence millions of short DNA fragments in each of the lanes. Typically, independent samples of mRNA are loaded into different lanes of the flow-cell such that sequencing reactions occur independently between samples. For illustration purposes, consider an example with seven subjects and seven treatment groups (T₁,...,T₇), where each subject is randomly assigned to one treatment group, and mRNA from each subject is loaded into a different lane (Figure 1). Notice that there is no biological replication because there is only a single subject in each treatment group.
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FIGURE 1. Hypothetical Illumina GA flow-cell with mRNA isolated from subjects within seven different treatment groups ($T_1, ..., T_7$) and loaded into individual lanes (e.g., the mRNA from the subject within treatment group 1 is sequenced in Lane 1). As a control, a $\Phi X$ genomic sample is often loaded into Lane 5. The bacteriophage $\Phi X$ genome is known exactly, and can be used to recalibrate the quality scoring of sequencing reads from other lanes (Bentley et al. 2008).

In order to analyze data from un-replicated designs, the sampling hierarchy must be taken into account. Regardless of the design, we can define three levels of sampling at work in RNA-Seq data: subject sampling, RNA sampling, and fragment sampling. Subjects (e.g., organisms or individuals) are ideally drawn from a larger population to which results of the study may be generalized (un-replicated data consists of a single subject within each treatment group). RNA sampling occurs during the experimental procedure when RNA is isolated from the cell(s). Finally, only certain fragmented RNAs that are sampled from the cells(s) are retained for amplification, and since the sequencing reads do not represent 100 percent of the fragments loaded into a flow-cell, fragment-level sampling is also at play.

Un-replicated data consider only a single subject per treatment group. Typically either there is one subject to which every treatment is applied (e.g., in Marioni et al. (2008), liver and kidney samples were extracted from one human cadaver), or one distinct subject within each treatment group (e.g., Figure 1). In either situation, it is not possible to estimate variability within treatment group, and the analysis must proceed without any information regarding within-group
TABLE 1
A 2x2 contingency table of (un-replicated) digital gene expression (DGE) measures for testing differential expression between Treatment1 and Treatment2 of Gene A. The cell counts $n_{ki}$ represent the DGE count for Gene A ($k = 1$) or the Remaining Genes ($k = 2$) for Treatment$i$, $i=1,2$. The $k^\text{th}$ marginal row total is denoted $N_k$, $N_i$ is the marginal total for column $i$, $N$ is the grand total.

<table>
<thead>
<tr>
<th></th>
<th>Treatment1</th>
<th>Treatment2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gene A</td>
<td>$n_{11}$</td>
<td>$n_{12}$</td>
<td>$N_1$</td>
</tr>
<tr>
<td>Remaining Genes</td>
<td>$n_{21}$</td>
<td>$n_{22}$</td>
<td>$N_2$</td>
</tr>
<tr>
<td>Total</td>
<td>$N_1$</td>
<td>$N_2$</td>
<td>$N$</td>
</tr>
</tbody>
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biological variation. As such, in the context of RNA-Seq, statistical methods for finding differences between groups are limited to RNA and fragment-level sampling information.

Since the sampling scheme for RNA-Seq is similar to SAGE (Velculescu et al. 1995), and there is a sizable statistical literature already established for analyzing differential DGE measures from un-replicated SAGE data, similar methods can be used for un-replicated RNA-Seq data. See Man et al. (2000), Romualdi et al. (2001), and Ruijter et al. (2002) for reviews and comparisons of techniques, and Tino (2009) for further discussion. For both RNA-Seq and SAGE data the analysis usually proceeds on a gene-by-gene basis by organizing the data in a 2x2 table (Table 1). Perhaps the most natural test for differential expression in the un-replicated case is Fisher’s Exact Test (Fisher 1935b) which fixes the marginal totals of the 2x2 table and tests differential expression using the hypotheses:

$$H_0 : \theta = 1 \text{ versus } H_A : \theta \neq 1,$$

where

$$\theta = \frac{\pi_{11}\pi_{22}}{\pi_{12}\pi_{21}},$$

and

$\pi_{kj}$ is the true proportion of counts in cell $k,i$ ($k = 1,2; i = 1,2$), assuming every transcript was isolated and perfectly sequenced. In Table 1 we can think of having $N_1$ white balls and $N_2$...
FIGURE 2. The log₂ fold change, between Treatment₁ and Treatment₂, of the normalized gene expression is plotted on the y-axis, and the mean log₂ expression is plotted on the x-axis. Gene expression counts were normalized by the column totals of the corresponding 2x2 table (e.g., Table 1). Blue dots represent significantly differentially expressed genes as established by Fisher’s Exact Test; grey dots represent genes with similar expression. The red horizontal line at zero provides a visual check for symmetry.

black balls in an urn. If we draw $N_1$ balls from the urn, we may ask, “What is the probability of observing an outcome at least as unlikely as $n_{i1}$ white balls?” If this probability (i.e., the P-value from Fisher’s Exact Test) is small, then the column classification has affected the draw from the urn. In our application, Gene A is differentially expressed between Treatment₁ and Treatment₂. One method of calculating two-sided P-values is to sum the probabilities of all 2x2 tables with probabilities less than or equal to that of the observed table where the probability of a 2x2 table (e.g., Table 1) is:
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\[ P = \frac{N_1 \cdot N_2 \cdot !N_1 \cdot !N_2 \cdot !}{N \cdot \tilde{n}_{11} \cdot !\tilde{n}_{12} \cdot !\tilde{n}_{21} \cdot !\tilde{n}_{22} \cdot !} \]  
(2)

where \( \tilde{n}_{ik} \) denotes the observed value of \( n_{ik} \). Note that there are several methods for computing two-sided P-values from Fisher’s Exact Test (Agresti 2002). Figure 2 illustrates the behavior of Fisher’s Exact Test for testing differential expression, between two treatment groups, for every gene in an RNA-Seq data set. It is worth remembering that Fisher’s Exact Test becomes more conservative as expression values decrease to zero, a point concurrent with the fact that genes with small expression values also demonstrate larger variability. This phenomenon as related to RNA-Seq data is discussed in detail in Bloom et al. (2009). The methods used by Kal et al. (1999) (a test of the equality of two binomial proportions) and Audic and Claverie (1997) (a Bayesian model with a Poisson likelihood and a flat prior on the mean) may also be used, although comparisons between these, and other, approaches to Fisher’s Exact Test (Man et al. 2000; Romualdi et al. 2001; Ruijter et al. 2002) show marginal differences in performance.

**Limitations of un-replicated data:** The fundamental problem with generalizing results gathered from un-replicated data is a complete lack of knowledge about biological variation. As Fisher (1935a) noted, without an estimate of variability (i.e., within treatment group), there is no basis for inference (between treatment groups). Although we can test for differential expression between treatment groups from un-replicated data, the results of the analysis only apply to the specific subjects included in the study (i.e., the results can not be generalized). To better understand the unrealistic conclusions that can be drawn from un-replicated data, suppose that an alien visits Earth and only observes two people, one male named “John” standing 177 cm and one female named “Jane” standing 180cm. The same reasoning that results in unrealistic conclusions from testing differential expression between treatment groups based on un-replicated
FIGURE 3. A multiple flow-cell design based on three biological replicates within seven treatment groups. There are three flow-cells with eight lanes per flow-cell. The control $\Phi X$ sample is in Lane 5 of each flow-cell. $T_{ij}$ refers to the $j^{th}$ replicate in the $i^{th}$ treatment group ($i = 1, ...., 7; j = 1,...,3$).

Data would compel the alien to believe that not only is John shorter than Jane, but that women are, on average, taller than men.

**Replicated data:** Consider extending the example illustrated in Figure 1 (i.e., seven treatment groups with one subject per treatment) to include two more biological subjects within each treatment group (Figure 3). The biological replicates allow for the estimation of within-treatment group (biological) variability, provide information that is necessary for making inferences between treatment groups, and give rise to conclusions that can be generalized.

A simple method for testing differential expression that incorporates within-group (or, within-treatment) variability relies on a Generalized Linear Model (GLM) with over-dispersion. The model is similar to the one provided by Lu et al. (2005). Consider a per-gene Poisson GLM. If $Y_{ijk}$ represents the DGE measure for the $j^{th}$ replicate ($j = 1, ..., J$) in the $i^{th}$ treatment group ($i = 1, ..., I$) of gene $k$, and $c_{ij}$ represents the overall number of reads from the $j^{th}$ replicate in the $i^{th}$ treatment group (e.g., $c_{ij} = \sum_k Y_{ijk}$), then we can model $Y_{ijk}$ as a Poisson($\mu_{ijk}$) random variable, where $\mu_{ijk} = \lambda_{ijk} c_{ij}$ and $\lambda_{ijk}$ represents the rate at which reads from the $j^{th}$ replicate in the $i^{th}$ treatment group map to the $k^{th}$ gene relative to all the other genes (Marioni et al. 2008). In this
example, the inclusion of the \( c_{ij} \) term is equivalent to normalizing by the total number of reads per lane, a common practice in RNA-Seq (Mortazavi et al. 2008). The following model is fit independently to each of \( k \) genes:

\[
\log \mu_{ijk} - \log c_{ij} = \alpha_k + \tau_{ik},
\]

where \( \alpha_k \) is the mean rate of expression across treatments for gene \( k \) and \( \tau_{ik} \) is the effect of the \( i^{th} \) treatment on the overall mean rate of expression for gene \( k \). Differential expression of gene \( k \) between treatment group \( i \) and treatment group \( i' \) is tested with the following hypotheses:

\[
H_0: \tau_{ik} = \tau_{ik}' \quad \text{versus} \quad H_A: \tau_{ik} \neq \tau_{ik}'
\]

A simple Poisson GLM assumes that \( \text{Variance}(Y_{ijk}) = \text{Mean}(Y_{ijk}) \), and when this assumption holds the hypotheses (4) can be tested by comparing the likelihood ratio test statistic (LRT) to a \( \chi^2_{df=1} \) distribution, where the LRT takes the form:

\[
\text{LRT} = 2 \sum_{i,j} y_{ijk} \log(\hat{\mu}_{ijk} / \hat{\mu}_{ijk}'),
\]

and \( \hat{\mu}_{ijk} \) is the maximum likelihood estimate (MLE) of \( \mu_{ijk} \) (under the alternative hypothesis) while \( \hat{\mu}_{ijk} \) is the MLE of \( \mu_{ijk} \) under \( H_0 \). If there is any within treatment group variability between individuals that is beyond that expected by Poisson sampling, then the required assumption of the Poisson GLM (i.e., that \( \text{Variance}(Y_{ijk}) = \text{Mean}(Y_{ijk}) \)) will not hold. In fact, if the LRT in (5) is used to evaluate the hypotheses in (4) when the assumption is violated, it typically results in inflated Type I error rates. In order to maintain the Type I error at the appropriate level we need to estimate a dispersion parameter \( \phi \), where \( \phi = \frac{\text{Var}(Y_{ijk})}{E(Y_{ijk})} \) (if \( \phi > 1 \) the data are said to be “over-
dispersed," in more rare cases, $\phi < 1$ and the data are “under-dispersed). The estimate of $\phi$

suggested in Faraway (2006), and Tjur (1998) is:

$$\hat{\phi} = \frac{\sum_{i,j} (y_{ijk} - \hat{\mu}_{ijk})^2 / \hat{\mu}_{ijk}}{m - p}$$

(6)

where $m$ is the total number of observations and $p$ is the number of estimated parameters in

model (3) (i.e., in our example with three replicates per treatment, $m = 3 \times 7$ and $p = 7$ ). Both

Faraway (2006) and Tjur (1998) argue that when over-dispersion is present, the hypotheses in (4)

should be tested by comparing the following test statistic to a $F_{df_{i=1}, df_{i=m-p}}$ distribution:

$$\frac{\text{LRT}}{\hat{\phi}}.$$  

(7)

This method is similar to the one described in Baggerly et al. (2004) where an over-dispersed

logistic regression model is fit to SAGE data to test differential expression. Other methods from

the SAGE literature may also be considered, for example Vencio et al. (2004) took a Bayesian

approach with a beta-binomial model accounting for within-class variability. Thygesen and

Zwinderman (2006) used a Poisson model with a gamma prior in an attempt to model all genes

simultaneously. Robinson and Smyth (2007, 2008) have incorporated the moderated test statistic

approach (Smyth 2004) into a negative binomial model to account for both within-class and

across-gene variability. Their approach is available in the edgeR package (Robinson et al. 2010)

from Bioconductor (Gentlemen et al. 2004).

**BALANCED BLOCK DESIGNS**

Without careful planning an unblocked design faces a fundamental problem with

generalizing the results, namely, the potential for confounding. With respect to RNA-seq

analysis, if the treatment effects are not separable from possible confounding factors, then for
any given gene, there is no way of knowing whether the observed difference in abundance between treatment groups is due to the biology or the technology (e.g., amplification or sequencing bias). For example in Figure 3 all replicates of Treatment\textsubscript{1} are sequenced in Lane 1 and all replicates of Treatment\textsubscript{2} are sequenced in Lane 2. Any differences in expression between Treatment\textsubscript{1} and Treatment\textsubscript{2} are confounded with possible lane effects that may persist across flow-cells. In fact, once the data are collected there is no way of separating the effects due to lane from the effects due to true treatment differences.

In RNA-Seq data, the design is the same for every gene, even though different genes have different variances and are potentially subject to different errors and biases. Of course, there are sources of variation that affect the majority of genes and these should certainly be integrated into the design. However, to ensure a robust analysis across all genes, sources of variation affecting only a minority of genes should be integrated into the design as well (e.g., a PCR based GC bias may only affect a small proportion of transcript fragments, therefore if it is possible, PCR batch should be integrated into the design). As such, we examine two main sources of variation (beyond the sampling hierarchy explained previously) that may contribute to confounding of effects in RNA-Seq data, namely “batch effects” and “lane effects”. Batch effects include any errors that occur after random fragmentation of the RNA until it is input to the flow-cell (e.g., PCR amplification and reverse transcription artifacts). Lane effects include any errors that occur from the point at which the sample is input to the flow-cell until data are output from the sequencing machine (e.g., systemically bad sequencing cycles and errors in base-calling).

Batch and lane effects have both been observed in previous studies. PCR amplification and reverse transcription artifacts were found to be non-negligible in both Balwierz \textit{et al.} (2009) and Chepelev \textit{et al.} (2009). Chepelev \textit{et al.} (2009) also observed systematically bad sequencing
cycles, and Rougemont et al. (2009) discusses the presence of base-calling errors in the Solexa platform. Although Marioni et al. (2008) found that variation across lanes generally follows a Poisson sampling process, they did observe considerably more variation for a non-negligible number of genes (on the order of $10^2$).

**Balanced blocks by multiplexing:** To eliminate confounding caused by batch or lane effects, consider the situation in which all samples of RNA are pooled into the same batch and then sequenced in one lane of a flow-cell. This would ensure that any batch effects are the same for all samples, and since the sequencing reaction is contained in one lane, all effects due to lane will be the same for all samples. While indeed this is an idealized situation, it can be accomplished by bar-coding the RNA immediately after fragmentation. Once bar-codes are attached to the random fragments, the samples can be pooled and processed together through the reverse transcription, size selection, and amplification steps. Typically, each lane is dedicated to sequencing one sample, so the number of samples $m$ is equal to the number of lanes $L$, ($m = L$). In order not to lose sequencing depth compared to the typical layout, $m$ total bar-coded samples can be pooled and processed together through the amplification step. The amplification product can then be divided into $L$ equal parts. Each part is then input to a different lane of the flow-cell. By exposing equal portions of every unique (i.e., bar-coded) sample to the same experimental conditions (i.e., same batch in the same lane), “balanced blocks” are formed.
FIGURE 4. Comparison of two designs for testing differential expression between treatments A and B. Treatment A is denoted by red tones, treatment B by blue tones. In the ideal balanced block design (left panel), six samples ($m = 6$) are bar-coded, pooled, and processed together. The pool is then divided into six equal portions that are input to six lanes ($L = 6$) of the flow-cell. Bar-coding in the balanced block design results in six technical replicates ($T = 6$) of each sample, while balancing batch and lane effects, and blocking on lane. The balanced block design also allows partitioning of batch and lane effects from the within-group biological variability. The confounded design (right panel) represents a typical RNA-Seq experiment and consists of the same six samples, no bar-coding, and does not permit partitioning of batch and lane effects from the estimate of within-group biological variability.
FIGURE 5 A Balanced Incomplete Block Design (BIBD) for three treatment groups (T₁, T₂, T₃) with one subject per treatment group (T₁₁, T₂₁, T₃₁) and two technical replicates of each (T₁₁₁, T₁₁₂, T₂₁₁, T₂₁₂, T₃₁₁, T₃₁₂). After fragmentation, each of the three samples are bar-coded and divided in two (e.g., T₁₁ would be split into T₁₁₁ and T₁₁₂), then pooled and sequenced as illustrated (e.g., T₁₁₁ is pooled with T₂₁₂ as input to Lane 1).

It is worth noting that if $L$ lanes are utilized, there is no loss of sequencing depth compared to running each sample in a lane. Figure 4 shows a comparison of this design to a typical design that confounds sample with batch and lane.

**Balanced Incomplete Block Designs and Blocking Without Multiplexing:** Although the previous balanced block design is convenient for illustration purposes, in reality resources, technical constraints, and the scientific hypotheses under investigation will dictate the number of treatments ($I$), the number of biological replicates per treatment ($J$), the number of unique bar-codes ($s$) that can be included in a single lane, and the number of lanes available for sequencing ($L$). When the number of unique bar-codes in one lane is less than the number of treatments (i.e., $s < I$) a complete block design (Figure 4) is not be possible. In these cases, we suggest using a Balanced Incomplete Block Design (BIBD). If $T$ is the total number of possible technical replicates per biological replicate, then a BIBD (according to our scheme of blocking
FIGURE 6. A design based on three biological replicates within seven treatment groups. For each of the three flow-cells there are eight lanes per flow-cell, and a control (ΦX) sample in Lane 5. Tij refers to the jth replicate in the ith treatment group (i = 1,...,7; j = 1,...,3). In this design the flow-cells form balanced complete blocks, and the lanes form balanced incomplete blocks.

by batch and lane by multiplexing) satisfies $T = sL/JI$ (Oehlert 2000). For a situation where there are three treatments ($I = 3$), a single subject within each treatment group ($J = 1$), the ability to include two unique bar-codes ($s = 2$) within a lane, and three available lanes for sequencing ($L = 3$) (currently, Illumina advertises 12 different bar-codes in a single lane http://www.illumina.com/documents/products/datasheets/datasheet_sequencing_multiplex.pdf) a BIBD as illustrated in Figure 5 is possible. Extensive lists of other BIBDs are given in Fisher and Yates (1963) and Cochran and Cox (1957).

Clearly multiplexing is useful for generating technical replicates that are effective in blocking on lane and batch effects to reduce confounding. However, it is important to understand that technical replicates are no substitute for independent biological replication, and that for a sufficient number of biological replicates certain designs can accommodate lane and/or flow-cell as blocking factors. As an illustration of this flexibility, the design in Figure 3 can be re-arranged as a balanced complete block design where the blocks are flow-cells, and a balanced incomplete block design where the blocks are lanes (Figure 6).
Analyzing a balanced block design: The generalized linear model in (3) can be expanded to include known blocking factors. Consider Figure 6 where both lane and flow-cell form blocks. The model for this design is:

\[
\log \mu_{ijkl} - \log c_{ij} = \alpha_k + \tau_{ik} + \nu_{jk} + \omega_{lk}
\]  

(8)

where \( \nu_{jk} \) is the effect of the \( f^{th} \) flow-cell on the mean rate of expression for gene \( k \), and similarly \( \omega_{lk} \) is the effect of the \( l^{th} \) lane on the mean rate of expression for gene \( k \). This model can be fit on a gene-by-gene basis implicitly assuming gene-by-block interactions (if one is unwilling to make this assumption, model (8) can easily be modified to fit all genes simultaneously allowing for the estimation of global blocking factors which can be used as off-sets in a per-gene model (3)). Notice that model (8) separates the lane and flow-cell effects (i.e., technological variation) from the within-group biological variability. The hypotheses for testing differential expression between Treatment \( i \) and Treatment \( i' \) are as in (4), and the dispersion parameter (\( \phi \)) is estimated as:

\[
\hat{\phi} = \frac{\sum_{i,j,f,l} (y_{ijkl} - \hat{\mu}_{ijkl})^2 / \hat{\mu}_{ijkl}}{m - p}
\]  

(9)

where \( m = 21 \) and \( p = 15 \), and the LRT is:

\[
\text{LRT} = 2 \sum_{i,j,f,l} y_{ijkl} \log(\hat{\mu}_{ijkl} / \hat{\mu}_{ijkl}).
\]  

(10)

The F-statistic for testing differential expression is simply the LRT in (10) divided by \( \hat{\phi} \) as estimated in (9). Under the null hypothesis of no differential expression, this F-statistic is approximately distributed as a \( F_{df1=1, df2=m-p} \) random variable.
Analyzing a balanced block design with technical replicates: Consider the balanced block design in Figure 4. Let $Y_{ijkt}$ represent the DGE measure for the $t^{th}$ technical replicate ($t = 1, \ldots, 6$) of the $j^{th}$ biological replicate ($j = 1, \ldots, 3$) in the $i^{th}$ treatment group ($i = 1, \ldots, 2$) of gene $k$. Then

$$Y_{ijk} = \sum_t Y_{ijkt}$$

and $Y_{ijk}$ can be modeled as before with a Poisson($\mu_{ijk}$) random variable,

where $\mu_{ijk} = \lambda_{ijk} c_{ij}$ and $\lambda_{ijk}$ represents the rate at which reads from the $j^{th}$ replicate in the $i^{th}$ treatment group map to the $k^{th}$ gene relative to all the other gene (the offset term $c_{ij}$ no longer represents the total number of reads per lane, but the total number of reads in the $j^{th}$ replicate of
Design A is a biologically un-replicated unblocked design with one subject for treatment group $T_1$ ($T_{11}$) and one subject for treatment group $T_2$ ($T_{21}$). Design B is a biologically un-replicated balanced block design with $T_{11}$ split (bar-coded) into two technical replicates ($T_{111}, T_{112}$), and $T_{21}$ split into two technical replicates ($T_{211}, T_{212}$) and input to Lanes 1 and 2. Design C is a biologically replicated unblocked design with three subjects from treatment group $T_1$ ($T_{11}, T_{12}, T_{13}$), and three subjects from treatment group $T_2$ ($T_{21}, T_{22}, T_{23}$). Design D is a biologically replicated balanced block design with each subject (e.g., $T_{11}$) split (bar-coded) into six technical replicates (e.g., $T_{111}, ..., T_{116}$) and input to six lanes.

The $i^{th}$ treatment group summed over technical replicates, $c_{ij} = \sum_{k,l} Y_{ijkl}$. The model, hypotheses, estimation, and testing procedures are the same as in (3-7) with $y_{ijk}$ replacing $y_{ijk}$. This analysis strategy does not include lane as a blocking factor, therefore lane effects will not be partitioned from estimates of within-treatment group variability (since only one batch was used, batch-to-batch variation was removed from residual error and batch effects need not be included in the model). However, since lane effects are balanced across treatment groups the potential for
confounding on lane effects is eliminated. In order to accurately partition the lane effects from estimates of within-treatment group variability a repeated measures GLM (see Faraway 2006) with over-dispersion is necessary. To our knowledge, hypothesis testing in this paradigm is currently problematic and a point of future research.

SIMULATIONS

To evaluate the effectiveness of the proposed multiplexed designs, we compare a biologically un-replicated unblocked design (A); a biologically un-replicated balanced block design with technical replicates (i.e., multiplexing) (B); a biologically replicated (triplicate) unblocked design (C); and a biologically replicated (triplicate) balanced block design with technical replicates by multiplexing (D) in an experimental setting testing differential expression between Treatment 1 \( (T_1) \) and Treatment 2 \( (T_2) \) (Figure 7). Gene counts were simulated across treatment groups, and we compare the false positive rate (1-specificity) and the true positive rate (sensitivity) for each design.

**Data Simulation:** We fixed the total number of reads at \( c = 3,000,000 \) and the mean sampling rate for treatment group \( T_1 \) (denoted \( \lambda_1 \)) at four different values \( (10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}) \) which corresponds to gene counts on the order of \( 10^4, 10^3, 10^2 \), and \( 10^1 \), respectively (e.g., \( c \times 10^{-5} = 30 \)). We varied the mean \( \log_2 \) fold change (LFC), between \( T_1 \) and \( T_2 \) from -3 to 3 in increments of 0.25. The treatment group \( T_2 \) sampling rate was calculated as:

\[
\lambda_2 = 2^{\log_2(\lambda_1) \times \text{LFC}}.
\]  

(11)

Gene counts \( Y_{1j} \) and \( Y_{2j} \) were sampled according to Poisson \( (c\lambda_1) \) and Poisson \( (c\lambda_2) \) distributions, respectively \( (Y_{1j} \sim \text{Poisson}(c\lambda_1), Y_{2j} \sim \text{Poisson}(c\lambda_2)) \). In order to evaluate the effectiveness of
modeling with an over-dispersed Poisson GLM, we added Gaussian noise to each gene count rounding to the nearest integer:

\[ Y_{ij}'' = Y_{ij}' + \lfloor \epsilon_j \rfloor, \]

\[ \epsilon_j \sim N(0, \sigma = \nu / \psi) \]

\[ \psi = 5, 10, 15, 100 \]

\[ \nu = (n\lambda_1 + n\lambda_2) / 2. \]

Four different simulation settings are considered, batch effect and lane effect (S1); batch effect and no lane effect (S2); no batch effect and lane effect (S3); and no batch effect and no lane effect (S4). Batch effects were simulated by adding Gaussian noise to each noisy gene count \((Y_{ij}')\) and rounding to the nearest integer:

\[ Y_{ij}''' = Y_{ij}'' + \lfloor \epsilon_j^{Y_{ij}'} \rfloor, \quad \epsilon_j^{Y_{ij}'} \sim N(0, Y_{ij}' / 10). \]

No noise was considered in settings S3 and S4 (i.e., for settings S3 and S4, \(Y_{ij}' = Y_{ij}''\)). Lane effects were simulated by Poisson sampling from \(Y_{ij}'\) and \(Y_{ij}''\) at different rates varying between lanes

\[ \tilde{Y}_{ij} \sim \text{Poisson}(Y_{ij}' \delta_j) \]

\[ \delta_j \sim \text{Discrete Uniform}\{0.65, 0.8, 0.95\}. \]

For settings with no lane effect (S2 and S4), the Poisson sampling rates were held constant \((\delta_j = 0.8)\). Since designs B and D included technical replicates, we distributed the respective sampling rates with no loss of depth for the gene counts in each biological replicate.

For design (A) we tested for differential expression using Fisher’s Exact Test (1), setting both column totals of the 2x2 table to 3,000,000. For design (B) we fit a balanced block design with technical replicates model. We set the offset term to \(c\), and since this design does not have
any biological replicates (i.e., $J=1$) we did not estimate dispersion. The likelihood ratio was compared to a $\chi^2_{df=1}$ distribution. For design (C) we fit model (3) with $c$ as the offset, estimated dispersion as shown in (6), and used the F-test described in (7). For design (D) we fit a balanced block design that acknowledged the technical replicates with $c$ as the offset.

We ran 10,000 simulations under each setting (S1-S4), varying $\lambda$ and $\psi$. The false positive rate (i.e., Type I error rate) was calculated as the proportion of times a gene was declared to be differentially expressed when the LFC was zero. The true positive rate (i.e., statistical power) was calculated as the proportion of times a gene was determined to be differentially expressed in the correct direction when the LFC was not zero.

**Results:** Receiver Operating Characteristic (ROC) curves offer a useful way of comparing false positive rates with true positive rates. Using ROC curves the true positive rate is typically plotted on the vertical axis, and the false positive rate plotted on the horizontal axis. The resulting curves can be compared by fixing a false positive rate (Type I error rate) and contrasting the corresponding true positive rates (statistical power). If one ROC curve is always above another, this indicates its superiority in classifying genes as differentially expressed. The diagonal identity line indicates the performance of classifying a gene as differentially expressed using a completely random guess (e.g., guessing differential expression 90% of the time yields a 90% true positive and false positive rate).

The designs featuring independent replication (Designs C and D) demonstrate remarkably better performance than the un-replicated designs (Designs A and B) whenever there is non-negligible within treatment group biological variability (Figure 8, Figures S1 and S2) across simulation settings (S1-S4). Even when there is very small within treatment group biological variability (Figure S3), the replicated designs still outperform the un-replicated designs.
FIGURE 8. ROC curves for the within group variability setting $\psi = 5$. The x-axis represents the false positive rate and the y-axis represents the true positive rate. The four panels of the graph show results for each of the four simulation settings. The ROC curve for the unblocked un-replicated design (A) is in solid red, the blocked un-replicated design (B) is in dotted red, the unblocked replicated design (C) is in solid blue, and the blocked replicated design (D) is in dotted blue. The replicated designs always outperform the un-replicated designs, and whenever there is a batch effect or lane effect, the blocked designs outperform their unblocked counterparts.
TABLE 2
The false positive rates (at the 0.05 nominal level) for designs A–D considering four settings of $\lambda_i$ ($10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}$), and four simulation settings (batch effect and lane effect (S1), batch effect and no lane effect (S2), no batch effect and lane effect (S3), no batch and no lane effect (S4)).

<table>
<thead>
<tr>
<th>Sampling Rate ($\lambda_i$)</th>
<th>Design</th>
<th>Simulation Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>S1</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>A</td>
<td>0.9655</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.9524</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.0494</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>0.0476</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>A</td>
<td>0.8789</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.8456</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.0477</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>0.0506</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>A</td>
<td>0.6521</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.5551</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.0522</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>0.0538</td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>A</td>
<td>0.2662</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2407</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.0482</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>0.0488</td>
</tr>
</tbody>
</table>
To evaluate which designs upheld the typical 0.05 false positive rate, for each of the four different sampling rates ($\lambda_i = 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}$), we calculated the proportion of times a gene was declared differentially expressed (using a p-value cutoff of 0.05) when the LFC was zero (Table 2; $\psi = 5$). The replicated designs (C and D) maintained the nominal significance level (0.05) across the simulation settings demonstrating that designs featuring independent replication coupled with analyses that estimate within-group variability are robust to batch effects, lane effects, and extra-Poisson variability. The estimated false positive rates for the un-replicated designs (A and B) suffered under all simulation settings, especially for genes with larger expression values. Although the false positive rates tended to decrease slightly as batch and lane effects were removed, in the absence of a batch or lane effect, the Type I error rates were still 4 to 5 times larger than the nominal significance level for genes with the smallest level of expression ($\lambda_i = 10^{-5}$).

Interestingly, the blocked designs did not outperform the un-blocked designs in terms of the false positive rates (Table 2). However, across simulation settings, whenever a batch or lane effect is present, the blocked designs demonstrate distinguishably higher true positive rates than the unblocked designs. We speculate that there are two reasons for this result. First, the blocked designs are all included in the same batch such that batch to batch variation is removed from residual error. Second, even though we did not use lane as a blocking factor, lane effects were balanced across every biological replicate in the two treatment groups thereby reducing the chance that a lane effect would overly influence one treatment group and produce a misleading result (i.e., confounding). Partitioning the variation due to lane through a statistical model that
included blocks on lanes may further enhance the performance of the blocked designs by reducing the residual error.

**DISCUSSION**

Fisher (1935a) was right. Replication, randomization, and blocking are essential components of any well planned and properly analyzed design. RNA-Seq designs and analyses are no exception. Luckily, the current format and properties of the NGS platforms lend themselves nicely to the concepts of randomization and blocking. However, the decision to biologically replicate remains in the hands of the scientist.

Our purpose in writing this paper is to demonstrate that variability (which is dependent on the organism, laboratory techniques, and the biological factors under investigation) may be larger than expected, and if not estimated properly will negatively affect the results of any study. This variability is positively correlated with the magnitude of the overall variability between biological subjects and can be dealt with by employing statistical models that not only accommodate estimation of within-treatment group biological variability, but remain faithful to a nominal significance level. Indisputably, the best way to ensure reproducibility and accuracy of results is to include independent biological replicates (technical replicates are no substitute) and to acknowledge anticipated nuisance factors (e.g., lane, batch, and flow-cell effects) in the design.

For both replicated and un-replicated scenarios the proposed balanced block designs benefit from both the NGS platform design, as well as multiplexing. These designs are as good, if not better than, their unblocked counterparts in terms of power and Type I error, and are considerably better when batch and/or lane effects are present. Realizing of course that it is not possible to determine whether or not batch and/or lane effects are present *a priori*, we
recommend the use of block designs to protect against observed differences that are attributable to these potential sources of variation. Since we understand both the expense associated with block designs, and the concern of multiplexing, we offer some alternatives. Certainly, it is possible to avoid multiplexing if there are enough biological replicates and sequencing lanes that allow for designs that block on lane and/or flow-cell (see Figure 6). However, if resources are limited (i.e., one flow-cell) multiplexing offers an alternative that at the very least eliminates the potential for confounding of effects. Multiplexing and blocking aside, the bottom line remains the same, results from un-replicated data cannot be generalized beyond the sample tested (here, differential expression).

Even though the benefits of good designs far outweigh any drawbacks, we anticipate objections to the multiplexed designs related to cost, loss of sequencing depth, and bar-code bias. To mitigate these concerns we provide some reassurances. First, although increased cost may be a concern, the added cost and time to multiplex is negligible when compared to the overall time and resources required for RNA-Sequencing. Second, there may be additional concerns that multiplexing will result in an overall loss of sequencing depth. This will only be problematic if enough bar-codes are incorrectly identified such that the read counts for each gene is affected. Recently, Phillipe et al. (2009) estimated that, on the Illumina platform, the probability that a 20 base transcript tag contains one or more sequencing errors is 0.0048. Using this as an upper bound on the probability that the bar-code will contain one or more sequencing errors (the bar-code is six bases long), in a lane with 10,000 million usable sequencing reads and 10,000 different transcripts, miscalled bar-codes would result in an average loss of at most 4.8 reads from the read count per transcript. Third, while there may be technical problems with multiplexing, such as uneven coverage of the samples or bias in the base-calls adjacent to the
bar-code, as long as the problem with uneven coverage is consistent within each sample, normalization schemes that take into account lane- and sample-specific coverage can correct for this (e.g., dividing each gene count by the coverage over sample, or the coverage over lane). Read bias associated with bar-coding is not problematic if it affects all samples the same way. Specifically, a proper normalization scheme will be robust to bias as long as the problem is consistent within sample.

The design principles that are presented here are applicable to a variety of applications involving quantitative comparisons across samples and can be put into practice on every NGS platform as applied to RNA-Seq. However, for other applications (e.g., ChIP-Seq and Copy Number Variant studies, etc.) a clearly defined statistical model that fully accounts for sources of variation must be developed before specific details of the design can be described.

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LITERATURE CITED


Balwierz, P. J., P. Carninci, C.O. Daub, J. Kawai, Y. Hayashizaki et al., 2009 Methods for analyzing deep sequencing expression data: constructing the human and mouse promoterome with deepCAGE data. Genome Biology 10: R79.


FIGURE S1. ROC curves for the within group variability setting $\psi = 10$. The x-axis represents the false positive rate, the y-axis represents the true positive rate. The four panels of the graph show results for each of the four simulation settings. The ROC curve for the unblocked un-replicated design (A) is in solid red, the blocked un-replicated design (B) is in dotted red, the unblocked replicated design (C) is in solid blue, and the blocked replicated design (D) is in dotted blue. The replicated designs always outperform the un-replicated designs and whenever there is a batch effect or lane effect, the blocked designs outperform their unblocked counterparts.
FIGURE S2. ROC curves for the within group variability setting $\psi = 15$. The x-axis represents the false positive rate, the y-axis represents the true positive rate. The four panels of the graph show results for each of the four simulation settings. The ROC curve for the unblocked un-replicated design (A) is in solid red, the blocked un-replicated design (B) is in dotted red, the unblocked replicated design (C) is in solid blue, and the blocked replicated design (D) is in dotted blue. The replicated designs always outperform the un-replicated designs and whenever there is a batch effect or lane effect, the blocked designs outperform their unblocked counterparts.
FIGURE S3. ROC curves for the within group variability setting $\psi = 100$. The x-axis represents the false positive rate, the y-axis represents the true positive rate. The four panels of the graph show results for each of the four simulation settings. The ROC curve for the unblocked un-replicated design (A) is in solid red, the blocked un-replicated design (B) is in dotted red, the unblocked replicated design (C) is in solid blue, and the blocked replicated design (D) is in dotted blue. The replicated designs always outperform their un-replicated counterparts and whenever there is a batch effect or lane effect, the blocked designs outperform their unblocked counterparts.