STAT 517
Spring 2009
2nd midterm
Practice Exam
1. There are 7 balls in a box. An unknown number of these balls are white, and the rest are black.
   Anirban takes out 3 balls without replacement from the box, and shows his sister that 2 of the 3 balls are white.
   Derive, with proof, the MLE of the number of white balls in the box.
   14 points
2. Find the smallest sample size $n$ for which the type I error probability is $\leq .05$ and the power is $\geq .95$ at $\theta = 1$, for testing $H_0 : \theta = 0$ vs. $H_1 : \theta > 0$ in the $N(\theta, 9)$ distribution, and you reject $H_0$ for large values of the sample mean.

10 points
3. Let $X, Y, Z$ be iid $N(\mu, 1)$ random variables.

(a) Give an unbiased estimate of $\mu^2$.

(b) Give an unbiased estimate of $\mu^3$.

(c) Give an unbiased estimate of $\mathbb{E}(|X - Y|^5)$.

(d) What is the MLE of $\max\{\mu, 0\}$? If the true value of $\mu = 0$, what is the probability that the MLE of $\max\{\mu, 0\}$ is equal to 0?

(e) Is it possible to unbiasedly estimate $\max\{\mu, 0\}$? Prove your answer.

$2+2+4+4+3 = 15$ points
4. Consider the two sample $t$ confidence interval for the difference of two normal means when the variances are assumed equal. Find the expected length of this interval when the sample sizes are $m, n = 10$, and the common variance is $\sigma^2 = 4$.

12 points
5. On the basis of two iid observations from an Exponential distribution with mean $\lambda$, the null hypothesis $H_0 : \lambda = 1$ will be rejected against the alternative $H_1 : \lambda = 3$ if $\bar{X} > 2$.

(a) Find the Type I error probability of this test.

(b) Find the power of this test.

$5+5 = 10$ points