1. a) Is the length of the $t$ confidence interval for a mean a constant or a random variable?
b) Does the expected length of the $t$ confidence interval go to zero when $n \to \infty$ for normally distributed data?
c) Does the expected length of the $t$ confidence interval go to zero when $n \to \infty$ for data that are distributed as a Cauchy?  
Justify all your answers.

2. What is the minimum sample size needed to ensure that the length of the nominal 99% score confidence interval for an unknown proportion is $0.08$?  
Why is it necessary to say nominal?

3. Is the pooled estimate of the variance for the two sample $t$ confidence interval unbiased?  
Why exactly is the assumption of a common variance required?

4. Suppose we have one observation $X \sim N(\mu, 4)$. We want to test $H_0 : \mu = 1$ vs. $H_1 : \mu > 1$.  
a) Is the null hypothesis simple?  
b) Is the alternative hypothesis simple?  
c) Consider the specific test which rejects $H_0$ if $X > 2.5$, accepts $H_0$ if $X < 2$ and if $X$ is between 2 and 2.5, then it rejects $H_0$ provided an auxiliary fair coin toss results in a head.  
i) Compute the type I error rate of this test.  
ii) Compute the power of this test at $\mu = 1.01, 1.5, 3$.  
iii) Where do you think the power converges when $\mu \to \infty$? Do you think that that is where the power should converge when $\mu \to \infty$?