STAT 417 Dr. DasGupta

First Midterm Practice Exam

1. Suppose two independent observations X_1, X_2 are available, of which $X_1 \sim Poisson(\lambda)$, and $X_2 \sim Poisson(2\lambda)$.

a) Find, with proof, a sufficient statistic.

b) Give an unbiased estimate of λ based on the sufficient statistic. What is its variance?

2. A random variable $X \sim N(\mu^2, \mu), \mu > 0$.

a. Find explicitly the Fisher information function, assuming that it exists.

b. Is μ unbiasedly estimable? f so, give one unbiased estimate.

c. Find the Cramér-Rao lower bound explicitly.

d. Does your unbiased estimate in part b attain the Cramér-Rao lower bound? Prove it or disprove it.

3. Anirban's tiger has a certain homerange in the Happy Hollow forest. The tiger doesn't tell anyone what the home-range is. But he does venture the information that the homerange is circular in shape.

Prints of the tiger's paws have been located at six coordinates: (1,0), (2,3), (2.2, 3.1), (.5, .1), (-2, -1.2), (-2.5, 1.5).
Give sensible estimates for the center and size of the tiger's homerange using these paw prints.

4. True or False?(No proofs required)

a. If a UMVUE exists, it has to be unique.

b. The Fisher information function has to be a decreasing function of the underlying parameter.

c. The likelihood function is a random variable.

d. UMVUEs are always asymptotically unbiased.

e. The factorization theorem is due to Neyman.

f. In a k-parameter problem, the minimal sufficient statistic is k-dimensional.

g. Minimal sufficient statistics are unique.

h. A little bias is a worthwhile thing.