1. Let \( \{a_t \}_{t=1}^{\infty} \) be a white noise process with \( E[a_t] = 0 \) and \( \text{var}[a_t] = \sigma_a^2 \). Define \( z_t = \mu + a_t - .5a_{t-1} \). Find the mean, autocovariance, and autocorrelation of \( z_t \), and verify that \( \{z_t\}_{t=1}^{\infty} \) is stationary.

2. Let \( \{y_t\}_{t=1}^{\infty} \) be a stationary process with mean \( \mu_y \) and autocovariance \( \gamma_y(s) = \text{cov}[y_t, y_{t-s}] \). Define \( z_t = y_t - y_{t-1} \). Obtain the mean and autocovariance of \( \{z_t\}_{t=1}^{\infty} \) in terms of those of \( y_t \) and verify that it is stationary.

3. Let \( \{y_t\}_{t=1}^{\infty} \) and \( \{z_t\}_{t=1}^{\infty} \) be two stationary processes with means \( \mu_y \) and \( \mu_z \) and autocovariances \( \gamma_y(s) \) and \( \gamma_z(s) \), independent of each other. Find the mean and autocovariance of \( w_t = ay_t + bz_t \), where \( a \) and \( b \) are constants, and show that \( \{w_t\}_{t=1}^{\infty} \) is stationary.

4. Let \( a_i, b_i \) be independent r.v.’s with \( E[a_i] = E[b_i] = 0 \) and \( \text{var}[a_i] = \text{var}[b_i] = \sigma_i^2 \). Compute the mean and autocovariance of \( z_t = \sum_{i=1}^{m} (a_i \cos 2\pi\omega_i t + b_i \sin 2\pi\omega_i t) \) and show that it is stationary. [Hint: you may want to use the trigonometric identity \( \cos x \cos y + \sin x \sin y = \cos(x - y) \).]

5. Let \( \{a_t\}_{t=1}^{\infty} \) be a white noise process with mean 0 and variance \( \sigma_a^2 \). Define \( z_0 = 0, z_t = \phi z_{t-1} + a_t, t = 1, 2, \ldots \), where \( |\phi| < 1 \).

(a) Express \( z_t \) explicitly in terms of \( a_t \).

(b) Calculate the autocovariance \( \text{cov}[z_t, z_{t+s}] \) for \( s > 0 \), and show that for large \( t \), \( z_t \) is approximately stationary.

6. Observing \( z_1, \ldots, z_N \) from a stationary process with autocovariance \( \gamma_k \) and autocorrelation \( \rho_k = \gamma_k / \gamma_0 \). It is known that \( \text{var}[\bar{z}] = (\gamma_0/N)[1 + 2 \sum_{k=1}^{N-1} (1 - k/N) \rho_k] \).

(a) If \( \rho_k \to 0 \) as \( k \to \infty \), show that \( \text{var}[\bar{z}] \to 0 \) as \( N \to \infty \).

(b) Compare \( \text{var}[\bar{z}] \) with the following autocorrelations: (i) \( \rho_k = 0, k \neq 0 \); (ii) \( \rho_1 = .8, \rho_2 = .55, \rho_k = 0, k > 2 \).

7. Problem 2.1 in the text (p. 569 in 3rd ed; p.701 in 4th ed.), plus

(d) After inspecting the graphs in (a)-(c), do you think the series is stationary?

(e) Calculate and plot the sample ACF for lags up to 6.

(f) Assume \( \rho_k = 0, k > 2 \). Obtain approximate standard errors for \( r_1 \), \( r_2 \), and \( r_k \), \( k > 2 \).

(g) Assume \( \rho_k = 0, k > 2 \). Obtain approximate correlation between \( r_4 \) and \( r_5 \).

Due January 23, 2019