Show your work on ALL questions. Unsupported work will NOT receive full credit.

Decimal answers should be exact, or to exactly 3 decimal places. (Examples: if it is .25 use .25; if it is .0057891234 then use .006.)

You are responsible for upholding the Honor Code of Purdue University. This includes protecting your work from other students.

Please write legibly. If a grader cannot read your writing, NO credit will be given.

You are allowed the following aids: two one-page 8.5” x 11” handwritten (in your handwriting) cheat sheet, a scientific calculator, and pencils or pens. Cheat sheets with photocopied or printed information are not allowed and are subject to disciplinary action.

Instructors will not interpret questions for you. If you do have questions, wait until you have looked over the whole exam so that you can ask all of your questions at one time.

You must show your student ID (upon request), turn in your cheat sheet and sign the class roster when you turn in your exam to your instructor.

Turn off your cell phone before the exam begins.

<table>
<thead>
<tr>
<th>Question</th>
<th>Points Possible</th>
<th>Points Earned</th>
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<tbody>
<tr>
<td>1</td>
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1. Assume iPhone5s have an average lifetime of 36 months. Let K be the amount of time until Jeremy’s iPhone5 dies.

   (a) Write down the distribution of K and its parameter(s).  \textbf{2 points}

   (b) What is the probability that the lifetime of Jeremy’s iPhone5 will be between 2 and 4 years, inclusive?  \textbf{2 points}

   (c) Given that Jeremy’s iPhone5 has lasted 20 months already, what is the chance that his iPhone5 will last more than 60 months?  \textbf{3 points}

   (d) What is the 70th percentile of the amount of time until Jeremy’s iPhone5 dies?  \textbf{3 points}
2. Let $X$ denote a continuous random variable with Cumulative Distribution Function, CDF, as follows.

$$F(x) = \begin{cases} 
0 & \text{if } x < 2 \\
\frac{1}{16} x^2 + \frac{1}{8} x - \frac{1}{2} & \text{if } 2 \leq x \leq 4 \\
1 & \text{if } x > 4.
\end{cases}$$

(a) Derive the probability density function, PDF. 3 points

(b) Compute $P[2.5 < X < 3.5]$. 3 points

(c) Calculate $E[2X+3]$. 4 points
3. The Lakers and the Kings are going to play 20 preseason games in September 2013. Suppose in each game the Lakers will win with probability 0.6. Let $X$ denote the number of games the Lakers win among these 20 pre-season games.

(a) Write down the distribution of $X$ and its parameter(s). **2 points**

(b) What is the probability that the Lakers will win between 12 and 14 preseason games, inclusive? **3 points**

(c) Using what you know about probability, how many games do you expect the Lakers to lose among these 20 games? **3 points**

(d) Does $X$ have any approximate distributions? Why or why not? If it does, compute the approximate probability that the Lakers will win at least 14 games. Otherwise, compute the exact probability. **5 points**
4. The midterm scores in STAT 301 follow a normal distribution with a mean of 70 and a standard deviation of 8.

   (a) What is the probability that a randomly picked student has score higher than 85? 3 points

   (b) Given the fact that a randomly chosen STAT 301 student has a score of at least 75, what is the probability that this student scored less than 90? 5 points

   (c) Suppose the top 10% of the students will receive an A on this midterm. What is the cutoff for an A on this midterm? 3 points
5. Phone calls arrive at the switchboard at Chase Bank according to a Poisson Process with a rate of 2 calls per minute. Let $X$ denote the number of calls that arrive at the switchboard from 10:00 A.M. to 10:10 A.M.

(a) Write down the distribution of $X$ and its parameter(s). 2 points

(b) What is the probability that either 15 or 16 calls are received between 10:00 and 10:10? 3 points

(c) Given that the last phone call just came in, what is the probability that the next phone call arrives in the next 2 minutes? 3 points

(d) Given that 15 calls arrived between 11:00 and 11:05, what is the probability exactly 5 of these calls occurred between 11:00 and 11:02? 4 points
6. Sally has two versions of a gene that determines height (say, A and B), and is able to pass one of these genes to her child (with equal likelihood). If she passes gene A to her child, the child’s height will be Normally distributed with mean 170cm and standard deviation 10cm. If she passes gene B to her child, the child’s height will be Normally distributed with mean 180cm and standard deviation 10cm. For each of the following, you may use the Empirical Rule if appropriate.

(a) Suppose the child received gene A. What is the probability that the child’s height will be less than 190cm? 2 points

(b) Suppose it is unknown which gene the child received. What is the probability that the child will be taller than 190cm? Hint: a tree diagram may be very useful here. 4 points

(c) Given that Sally’s child grows to be taller than 190cm, what is the probability the child received gene A? Hint: your tree diagram from part (b) could also be used here. 2 points
7. The time Geoff takes to finish a bottle of Sprite is evenly distributed between 20 and 35 seconds, whereas it takes Tim equally likely between 15 and 45 seconds to finish a bottle of Sprite. Assume the time it takes one person to finish a bottle of Sprite is independent of the time it takes the other person to finish a bottle of Sprite. Let $G$ denote the time Geoff takes to finish a bottle of Sprite, and $T$ denote the time Tim takes to finish a bottle of Sprite.

(a) What is the probability that it takes Geoff less than 30 seconds to finish a bottle of Sprite knowing that it has taken him more than 25 seconds to finish? 3 points

(b) What is the probability that at least one of these people can finish a bottle of Sprite in less than 25 seconds? 4 points

(c) Geoff and Tim are racing to finish drinking their bottles of Sprite. Let $W$ be the event that Geoff finishes his Sprite before Tim. What is $P(W)$? Hint: The PMF table below might help you to solve this problem, though other approaches are acceptable as well. 3 points

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<td>$P(W</td>
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