Your Section (circle one):

Pan (7:30am)        Juan (8:30am)        Juan (9:30am)        Allison (10:30am)
Allison (11:30am)   Mike L. (12:30pm)   Mike C. (1:30pm)     Grant (2:30pm)
Grant (3:30pm)      Faye (4:30pm)

• Show your work on ALL questions. Unsupported work will NOT receive full credit.
• Decimal answers should be exact, or to exactly 4 decimal places. (Examples: if it is .25 use .25; if it is .57891234 then use .5789.)
• You are responsible for upholding the Honor Code of Purdue University. This includes protecting your work from other students.
• Please write legibly. If a grader cannot read your writing, NO credit will be given.
• You are allowed the following aids: a one-page 8.5” x 11” handwritten (in your handwriting) cheat sheet, a scientific calculator, and pencils or pens. Cheat sheets with photocopied or printed information are not allowed and are subject to disciplinary action.
• Instructors will not interpret questions for you. If you do have questions, wait until you have looked over the whole exam so that you can ask all of your questions at one time.
• You must show your student ID (upon request), turn in your cheat sheet and sign the class roster when you turn in your exam to your instructor.
• Turn off your cell phone before the exam begins.

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1. You are offered the following game. You draw one card from a well-shuffled deck of cards (4 suits, 13 face values). If it is the Ace of Hearts, you win $20. If it is any other Ace, you win $2. If it is any other Heart, you win $1. Nothing else pays. Let $X$ be the amount of money you win.

(a) What is the PMF for $X$? 4 points

(b) What is the expected value of $X$? 3 points

(c) What is the variance of $X$? 3 points

(d) Suppose it costs a certain amount of money to play this game. What amount of money should the game cost in order for it to be fair? (Recall: a game is fair if $E(X - \text{cost}) = 0$.) 2 points
2. Customers arrive at a convenience store at a rate of 12 per hour.

   (a) What is the distribution of the number of customers in the first three hours? 3 points

   (b) Find the probability that exactly 36 customers arrive in the first three hours. 3 points

   (c) What is the probability that exactly 12 customers arrive in each of the first three hours? 4 points

   (d) Why don't the answers in (b) and (c) match? 2 points
3. An American roulette wheel has 38 slots: 18 are red, 18 are black, and 2 are green. Suppose that you observe red on 5 consecutive spins.

   (a) Let $B$ be the number of additional spins until you observe green. Given that you’ve observed red on 5 consecutive spins, what is the distribution (and parameter(s)) of the number of additional spins needed until you observe green? 3 points

   (b) What is the expected number of additional spins it will take observe green? 2 points

   (c) What is the expected number of additional spins it will take to observe two greens? 2 points
4. Jimmy plays a cruel joke on his friends, one of which is named Dwayne. For Halloween, he allows them to pick candy randomly from his big bowl which contains 1,260 items, 12 of which are Baby Ruth bars. He allows them to pick 4 pieces of candy, but after each pick he makes them return it to the bowl. Let $B$ be the number of Baby Ruth bars sampled by a friend named Dwayne.

(a) Name the distribution and the value of the parameter(s) of $B$. 3 points

(b) Find the probability that $B = 5$. 3 points

(c) Find the probability that $B$ is an odd prime number (1 is not prime). If an approximation is appropriate, list the approximation, the reason(s) you can use it, and give the approximate probability instead of the exact probability. 4 points
5. Suppose Tanya wants to go fishing in her favorite river. This particular river has 400 fish, 100 of which are salmon. Tanya is quite skilled at fishing, and is able to catch 15 fish per hour (on average). For these questions, assume that Tanya keeps the fish she catches. Let $S$ be the number of salmon in Tanya’s sample.

(a) Suppose Tanya planned to catch exactly 30 fish. What is the distribution of the number of salmon in Tanya’s sample? 4 points

(b) Given the information in part (a), what is the expected number of salmon in Tanya’s sample? 2 points

(c) Suppose Tanya instead planned to fish for 2 hours and catch as many fish as she could. What is the distribution of the number of salmon in Tanya’s sample? 2 points

(d) Given the information in part (c), what is the expected number of salmon in Tanya’s sample? 3 points
6. There are 21 balls in a jar. 5 of them are blue, 6 of them are black, 3 of them are yellow, and 7 of them are red. John randomly samples 8 balls from the jar (without replacement). If he selects fewer than 2 black balls, his mom will give him a chocolate bar.

(a) What is the probability that John can get a chocolate bar in the first draw? 4 points

(b) John really wants to get 4 chocolate bars from his mom, so he repeatedly draws 8 balls (as above) until he gets his 4th chocolate bar. Let $X$ denote the number of draws needed until he gets the 4th chocolate bar. Name the distribution of $X$ and specify its parameter(s). 3 points

(c) How many draws can John expect to have to make until he gets the 4th chocolate bar? What is the variance in the number of draws needed until getting the 4th chocolate bar? 4 points
7. Mark is an avid table-top gamer, and has a 4-sided die with sides numbered 1 to 4. He rolls this die 3 times.

(a) What is the probability that the sum of three tosses was at most 4? (Hint: Try listing out the ways Mark can roll a sum of 4) 2 points

(b) What is the probability that among the three tosses Mark did not roll a '4'? 2 points

(c) What is the probability that among the three tosses Mark tossed at least one '1' and at least one '3'? (Hint: listing all of the possibilities is tedious, but possible. Try using set notation and rules instead.) 4 points

(d) Knowing that Mark has tossed three even numbers, what is the probability that the sum of the three tosses is 8? (Hint: Try listing out the ways Mark can roll a sum of 8 given he tossed only even numbers) 3 points