STUDY NOTES FOR LINEAR MODELS

XIONGZHI CHEN

1. Notations and Conventions

a: Unless otherwise specified, all notations bear their generic meanings as implied by the contexts.
b: Unless otherwise stated, all assumptions needed to induce or disprove the conclusions are automatically proposed, either explicitly or implicitly.

2. Two identities for Exponential Family

The exponential family in [1] is defined as
\[ f(y; \theta) = \exp \left((y\theta - b(\theta)) / a(\phi) + c(y, \phi)\right) \]
from which two identities are derived.

While I tend to regard the solution to those two identities (in the form of differential equations) as the exponential family. Anyway, an easy derivation comes hereunder:

(1) Since \( \int f dy = 1 \), then
\[
0 = \frac{\partial}{\partial \theta} \int f dy = \int \frac{\partial}{\partial \theta} f dy = \int y - b'(\theta) a(\phi) f dy = \frac{1}{a(\phi)} \left[ \int y f dy - \int b'(\theta) f dy \right]
\]

Suppose Lebesgue dominated convergence theorem (LDCT) holds in order to justify the interchange of the order of integration and differentiation.

(2) With the above assumption, then
\[
0 = \frac{\partial^2}{\partial \theta^2} \int f dy = \int \frac{\partial^2}{\partial \theta^2} f dy = \int -b''(\theta) a(\phi) f dy + \int \frac{[y - b'(\theta)]^2}{a^2(\phi)} f dy
\]

\[
= -\frac{b''(\theta)}{a(\phi)} + \frac{\text{Var}(Y)}{a^2(\phi)}
\]
References