Simultaneous Inference

- Consider a collection of confidence intervals
- Each interval has \((1 - \alpha)\)% confidence level
- What about the overall confidence level?
  - This is defined as the level of confidence that all constructed intervals contain their true parameter values
  - Often much lower than the individual \((1 - \alpha)\)% CI level
- We will adjust the individual confidence levels upward so the overall CI level is closer to \(1 - \alpha\)
- Recall confidence band \(\rightarrow\) widened individual intervals

Joint Estimation of \(\beta_0\) and \(\beta_1\)

- Will focus here on forming a joint rectangular region formed by the individual CIs
- If estimates were independent
  - Overall confidence level of rectangle is \((1 - \alpha)^2\)
  - Could set equal to 0.95 and solve for \(\alpha\)
- Estimates \((b_0, b_1)\) are not independent so how do we handle this?
- Given normal error terms, can show \((b_0, b_1)\) multivariate normal
- Can show natural (i.e., smallest) confidence region defined by an ellipse (STAT 524)
**Bonferroni Correction**

- Let $A_1$ denote event that CI excludes $\beta_0$
- Let $A_2$ denote event that CI excludes $\beta_1$
- By construction $\Pr(A_1) = \Pr(A_2) = \alpha$
- What is prob that both events don’t occur?

$$\Pr(A_1 \cap A_2) = 1 - \Pr(A_1 \cup A_2) = \Pr(A_1) + \Pr(A_2) - \Pr(A_1 \cap A_2) \leq \Pr(A_1) + \Pr(A_2)$$

- If $\Pr(A_1) + \Pr(A_2) = .05$, then $\Pr(A_1 \cap A_2) \geq 0.95$

**Mean Response CIs**

- Could apply Bonferroni correction
  - Want to know $E(Y|X)$ for $g$ $X$’s
  - Construct CIs using $\alpha^* = \alpha/g$
  - Reasonable approach when $g$ small

$$\hat{Y}_h \pm Bs(\hat{Y}_h) \text{ where } B = t(1 - \alpha/2g, n - 2)$$

- Previously discussed Working-Hotelling
  - Uses $F$ distribution instead of $t$ distribution
  - Coefficient does not change as $g$ increases

$$\hat{Y}_h \pm Ws(\hat{Y}_h) \text{ where } W^2 = 2F(1 - \alpha, 2, n - 2)$$

**Prediction Intervals**

- Could apply Bonferroni correction
  - Want to know $Y_{h(new)}$ for $g$ $X$’s
  - Construct PIs using $\alpha^* = \alpha/g$
  - Reasonable approach when $g$ small

$$\hat{Y}_h \pm Bs(\text{pred}) \text{ where } B = t(1 - \alpha/2g, n - 2)$$

- Can also use Scheffe’ procedure
  - Uses $F$ distribution instead of $t$ distribution
  - Coefficient increases as $g$ increases

$$\hat{Y}_h \pm Ss(\text{pred}) \text{ where } S^2 = gF(1 - \alpha, g, n - 2)$$
Regression through the Origin

- Many instances where theory suggests true population line should go through origin
- Statistical model under this restriction is
  \[ Y_i = \beta_1 X_i + \varepsilon_i \text{ where } \varepsilon_i \sim N(0, \sigma^2) \]
- Can show \( b_1 = \frac{\sum X_i Y_i}{\sum X_i^2} \) in this case
- In a sense you are forcing \( b_0 \) to be zero
- Model causes problems with \( R^2 \) and other statistics
- Little is lost fitting the intercept and slope in all cases
- **Note:** If no intercept, no adjustment necessary for family of tests

Measurement Error

- **Measurement Error in \( Y \)**
  - Generally not a problem provided error is random and unbiased
  - Error term in model represents unexplained variation which is often a combination of many factors not considered
- **Measurement Error in \( X \)**
  - Can cause problems
  - Often results in biased estimators (slope shrunk towards zero)
  - Reduces strength of association
  - Berkson error model: special case where predictor variable is set at a target level. This does not result in biased parameter estimates.

Inverse Predictions

- Given \( Y_h \), predict corresponding \( X, \hat{X}_h \)
- Given fitted equation this is
  \[ \hat{X}_h = \frac{Y_h - b_0}{b_1} \quad b_1 \neq 0 \]
- This is the MLE (i.e., function of \( b_0, b_1 \))
- Approximate CI can be constructed using inverse mapping of CI for \( \hat{Y}_h \)
  \[
  Y_h \pm t(1 - \alpha/2, n - 2)s(\hat{Y}_h) - b_0 \\
  \frac{b_1}{b_1} \\
  \hat{X}_h \pm t(1 - \alpha/2, n - 2)s(\hat{Y}_h)/b_1
  \]

Background Reading

- KNNL Chapter 4
- KNNL Chapter 5: Matrix Algebra