Blocking and Confounding in $2^k$ Factorial Designs

Confounding in $2^k$ with only 2 blocks

- Blocks assumed to allow $2^{k-1}$ combinations
- First consider $2^2$ factorial (2 combs per blk)
- Possible pairings
  1. (1) and b together → a and ab together
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1. Effect of A, $(ab+a-b-(1))/2$, is block difference
2. Effect of B, $(ab-a+b-(1))/2$, is block difference
3. Effect of AB, $(ab-a-b+1)/2$, is block difference

Both have a main effect confounded with block
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\(2^k\) Factorial in Four Blocks

- Four blocks each containing \(2^{k-2}\) EUs
- Useful in situations where \(k \geq 4\)
- Must select two effects to confound
- Will result in a third confounded factor
- Consider 6 factor factorial run in 4 blocks of 16 EUs
  - Block 1 uses \(ABC + \) and \(DEF +\)
  - Block 2 uses \(ABC + \) and \(DEF -\)
  - Block 3 uses \(ABC - \) and \(DEF +\)
  - Block 4 uses \(ABC - \) and \(DEF -\)
- Results in \((ABC)(DEF) = ABCDEF\) confounded
  - \(ABCD\) and \(DEF\) → \(ABCEF\) confounded
  - \(AB\) and \(ABEF\) → \(EF\) confounded
- Can extend to 8 and 16 blks
- Table 7-8 summarizes these designs (pg 298)

Partial Confounding

- Can replicate blocking design
- Confound different effects each replication
- Allows estimation of all effects
  - Confounded effects based on nonconfounded replicates
  - Can use Yates’ Algorithm for all nonconfounded effects
  - See Example 7-3 (pg 300)

\[
\text{proc glm;}
\]
\[
\text{class fact1 fact2 fact3 block;}
\]
\[
\text{model y= block fact1|fact2|fact3;}
\]

Example

Consider a \(2^3\) factorial run in 4 blocks

Each replicate will result in 3 confounded effects

Consider 4 replicates for 32 total observations

Replicate 1: Confound BC and AC → AB
Replicate 2: Confound BC and ABC → A
Replicate 3: Confound AC and ABC → B
Replicate 4: Confound AB and ABC → C

Three replicates to estimate A, B, and C
Two replicates to estimate AB, AC, and BC
One replicate to estimate ABC
**Data**

Replicate 1 - AB, AC, BC Confounded

<table>
<thead>
<tr>
<th>Blk 1</th>
<th>Blk 2</th>
<th>Blk 3</th>
<th>Blk 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>75 (1)</td>
<td>89 ab</td>
<td>61 a</td>
<td>30 b</td>
</tr>
<tr>
<td>100 abc</td>
<td>73 c</td>
<td>45 bc</td>
<td>54 ac</td>
</tr>
</tbody>
</table>

Replicate 2 - A, BC, ABC Confounded

<table>
<thead>
<tr>
<th>Blk 1</th>
<th>Blk 2</th>
<th>Blk 3</th>
<th>Blk 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 (1)</td>
<td>47 a</td>
<td>1 b</td>
<td>26 ac</td>
</tr>
<tr>
<td>34 bc</td>
<td>81 abc</td>
<td>35 c</td>
<td>52 ab</td>
</tr>
</tbody>
</table>

Replicate 3 - B, AC, ABC Confounded

<table>
<thead>
<tr>
<th>Blk 1</th>
<th>Blk 2</th>
<th>Blk 3</th>
<th>Blk 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>58 (1)</td>
<td>48 a</td>
<td>18 b</td>
<td>68 ab</td>
</tr>
<tr>
<td>42 ac</td>
<td>52 c</td>
<td>82 abc</td>
<td>32 bc</td>
</tr>
</tbody>
</table>

Replicate 4 - C, AB, ABC Confounded

<table>
<thead>
<tr>
<th>Blk 1</th>
<th>Blk 2</th>
<th>Blk 3</th>
<th>Blk 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>47 (1)</td>
<td>34 a</td>
<td>50 c</td>
<td>37 ac</td>
</tr>
<tr>
<td>57 ab</td>
<td>4 b</td>
<td>80 abc</td>
<td>27 bc</td>
</tr>
</tbody>
</table>

Since no effect is estimated from all replications (except error), we will compute the effect sums associated with each replicate and combine the appropriate information.

---

**SAS Output**

```sas
options nocenter ps=50 ls=80;

data new;
  input repl blk a b c resp;
  cards;
  110007 5
  111111 0 0
  121108 9
  120017 3
  ... 
  430015 0
  431118 0
  441013 7
  440112 7
  ;

proc glm;
  class repl blk a b c;
  model resp = repl blk(repl) a|b|c;
```

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type I SS</th>
<th>Mean Square</th>
<th>F Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>REPL</td>
<td>3</td>
<td>3040.093750</td>
<td>1013.364583</td>
<td>36481.13</td>
</tr>
<tr>
<td>BLK(REPL)</td>
<td>12</td>
<td>7568.375000</td>
<td>630.697916</td>
<td>22705.13</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>2420.041667</td>
<td>2420.041667</td>
<td>87121.50</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0.0416667</td>
<td>0.0416667</td>
<td>1.50</td>
</tr>
<tr>
<td>A*B</td>
<td>1</td>
<td>3600.000000</td>
<td>3600.000000</td>
<td>99999.99</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>100.041667</td>
<td>100.041667</td>
<td>3601.50</td>
</tr>
<tr>
<td>A*C</td>
<td>1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.00</td>
</tr>
<tr>
<td>B*C</td>
<td>1</td>
<td>400.000000</td>
<td>400.000000</td>
<td>14400.00</td>
</tr>
<tr>
<td>A<em>B</em>C</td>
<td>1</td>
<td>0.125000</td>
<td>0.125000</td>
<td>4.50</td>
</tr>
</tbody>
</table>

---

24-8